3C-GRAM BOOK NO. 1

an analysis of some

GAMES *of* FUN & STRATEGY

BY THE MATHEMATICAL
SERVICES DEPARTMENT OF
COMPUTER CONTROL CO., INC.

INTRODUCTION

This booklet has been assembled by the technical staff of the Mathematical Services Department of 3C from material which they had previously prepared and presented with less formality under the title of 3C GRAMS. It is offered as a source of diversion and recreation to today's over-worked scientific toilers and executives in the fields of mathematics and machine computation. It is also offered with less altruistic motivation as a means of acquainting these people and others with the capabilities possessed by 3C for assuming some or all of the burdens of work requiring mathematical analysis, programming, and/or machine computation. These capabilities are discussed in the final section of this booklet.

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In Japan, it was said that GO was forbidden to proletarians and women, because it wasted too much of a laborer's time, and women—well, women were far too cunning already. Be that as it may, GO still retains much of its early aristocratic flavor, and is regarded as one of the necessary refinements of the civilized Japanese man. It originated in China in antiquity—at least two thousand years ago and perhaps a great deal earlier than that—and is probably the oldest of all known games. In Japan, where it has been played with surpassing skill for eleven centuries, it is looked upon as the national pastime. For all its surface simplicity, GO is comparable, if not superior, to chess in intellectual challenge. It has often been called the chess of the Orient, and like chess it is a war game of pure skill between two opponents.

GO is played on a 17½ inch by 16 inch grid, as shown in *Figure 1*, with small circular pieces called "stones". (Small dots mark the position of handicapped stones, see page 6). There are 361 points of intersection ("points") on the grid, and 361 matching stones, 181 black and 180 white, which are played on these intersection points. The primary object of the game is to surround vacant intersection points, or "territory." In this process stones may be captured, and the player with the highest score, which is calculated by counting both captured intersection points and captured stones, is the winner. Provided there is no handicap (see page 6) the weaker of the two players is given the black stones and moves first by placing a stone on any vacant intersection. A stone once played cannot be moved; it stays where it is until the game ends or it is captured. The two players alternately place a stone on any vacant point until the entire board is covered or until both players decide that nothing can be gained by continuing the play, and come to an agreement on each other's score. It is rarely necessary to use all the stones to complete a game.

In scoring, only the vacant points are counted as territory, regardless of how many stones it may have cost to surround a point. Thus, whether a player uses three or eight stones to capture one intersection, he scores one point. A stone is captured and removed from the board when all the horizontal and vertical points immediately adjacent to the stone have been occupied by enemy stones, as shown in *Figure 2*. When vacant, adjacent

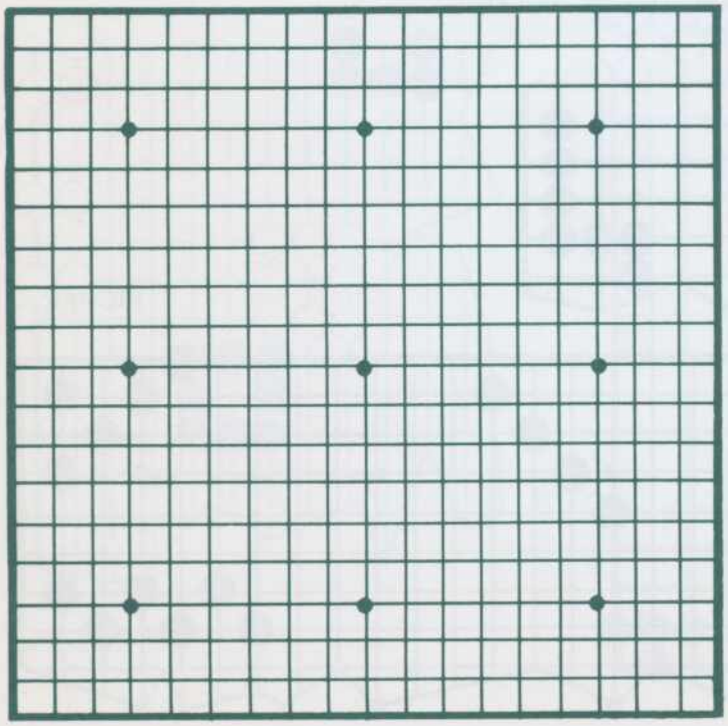


FIGURE 1

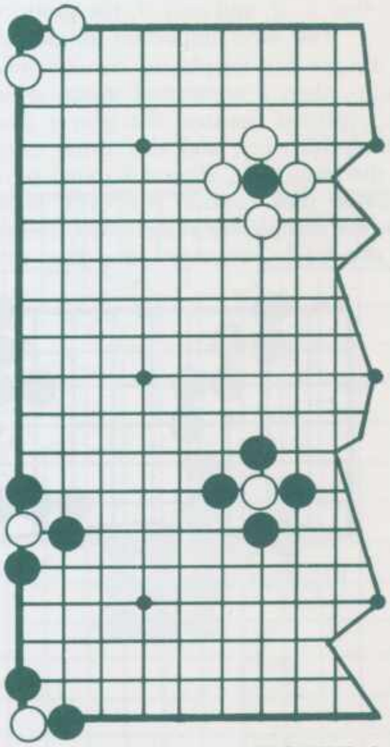


FIGURE 2

points are called "liberties." When a stone or a group of connected stones has only one remaining liberty, it is "in check." Note that it is not necessary to occupy the adjacent *diagonal* points, to capture the stone. Two or more stones of the same color can be connected into a single unit by placing them on adjoining horizontal and vertical intersections. Again, note that diagonal adjacent points are not considered adjoining intersections. *Figures 3 and 4* will clarify this. Each of the four groups in *Figure 3* is connected, and each of the three groups in *Figure 4* is not connected. Connected stones "die" or "live"—i.e., are or are not captured—as a group. They are captured and removed from the board when all the horizontal and vertical points immediately adjacent to all the connected stones have been occupied by stones of the opposite color. As with a single stone, a group of stones is in check when it has only one remaining liberty. *Figure 5* illustrates this point: each group is in check and will be captured if the player plays in the point marked A.

Neither player is allowed to play a stone on a vacant intersection point which is surrounded horizontally and vertically by enemy stones, unless the placing of this stone captures one or more of the surrounding enemy stones. In *Figure 6* the placing of a black stone at A is legal, but the placing of a stone at B would not be legal. A vacant intersection point which is completely surrounded horizontally, vertically and diagonally by stones of the same color is called an "eye." *Figure 7* illustrates some typical eyes. Since a player is not allowed to play on an eye unless by so doing he captures one or more of the surrounding stones, it follows that an eye can be played on only if all the outside liberties have already been played on: that is, if, and only if, the group which forms the eye is in check.

The most important single principle of the game follows logically. If an eye can be played on only when it is the last remaining liberty of a group, then a connected group which has two separated eyes can *never* be captured because the player can place only one stone on the board during his play, and this stone cannot be placed in two different places at the same time. *Figure 8* shows some groups which can never be captured because they contain two eyes. Note that even though some of the eyes are not completely surrounded, the pieces forming the eyes are connected, or effectively connected, and thus cannot be captured.

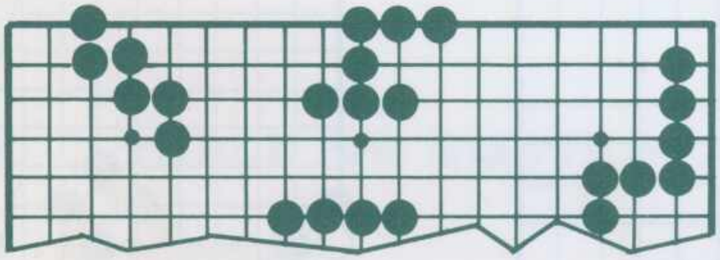
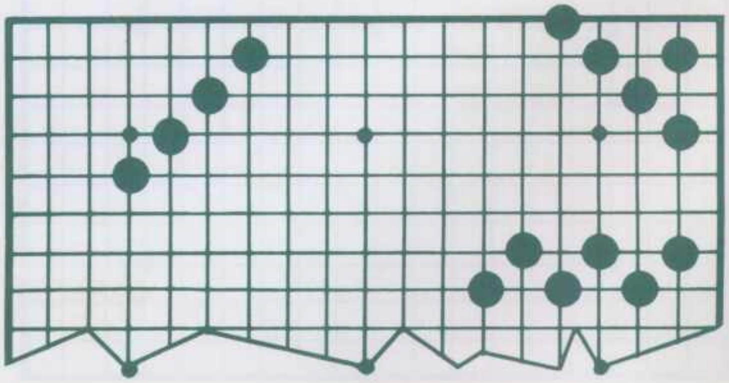


FIGURE 3

FIGURE 4



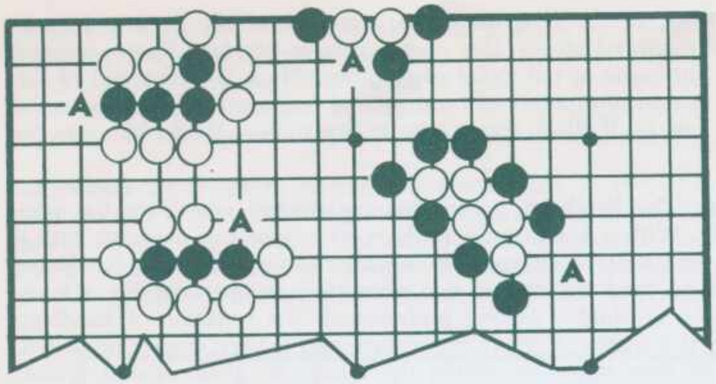


FIGURE 5

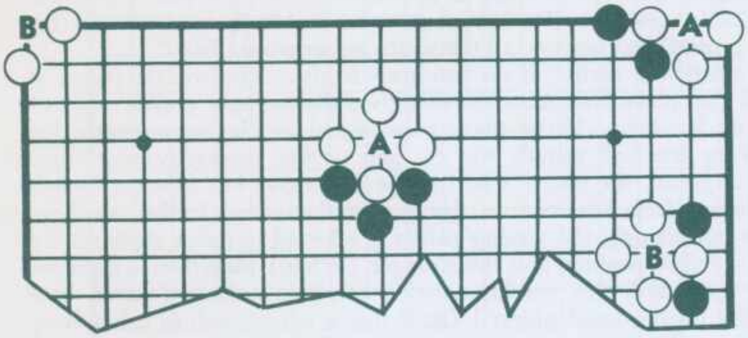


FIGURE 6

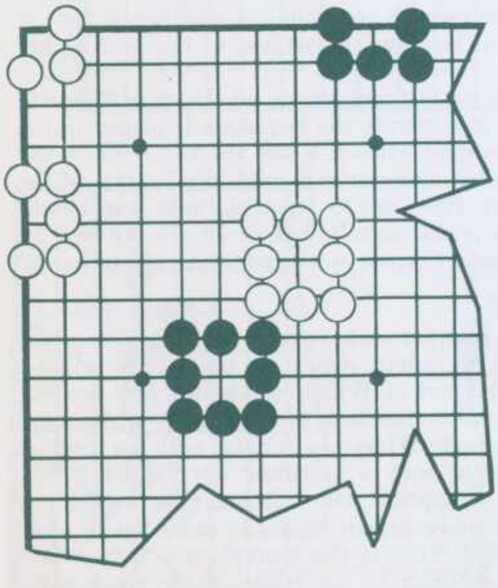
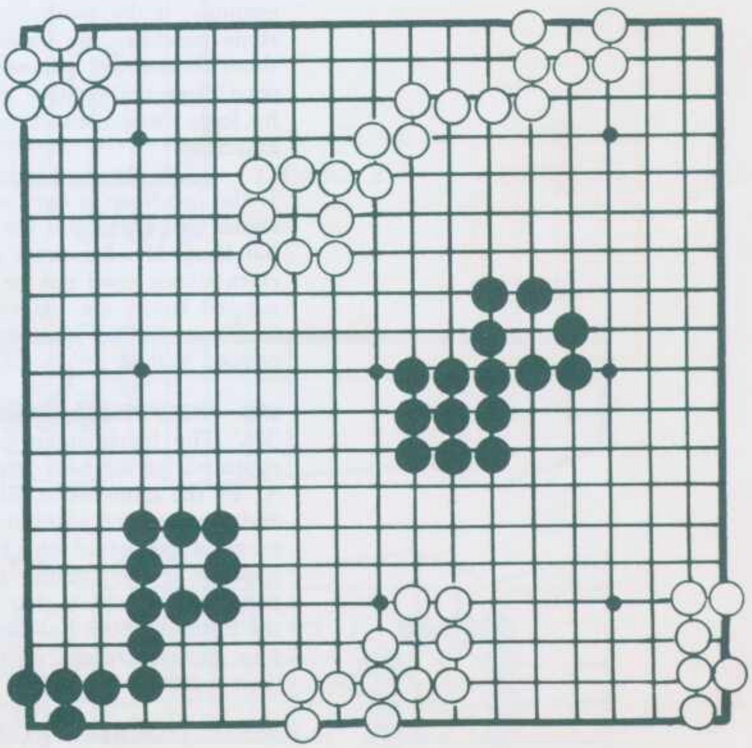


FIGURE 7

FIGURE 8



While, as noted above, the primary purpose of the game is to gain territory, the captured stones also play a role in scoring. Each captured stone counts the same as one point of territory. Thus, if at the end of the game White has twenty points of territory and has captured ten stones, he scores thirty points. If Black scores twenty-five points, White has won by five points.

HANDICAPS: The handicap system has contributed greatly to the popularity of GO. With a handicap, a fairly weak player can compete with a superior one and achieve an exciting and interesting game. GO handicaps are well defined, and the system can accurately compensate for a fairly wide difference in ability. Among professionals the question of handicap arrangements is a matter of great importance and is very strictly regulated. A handicap of four stones is usually the maximum given among professionals, and if used between players of the same degree, the player who is spotted four stones would probably win by at least thirty-six points. Beginners may be spotted as many as seventeen stones, but this is practical only with rank amateurs. If a seventeen-stone handicap were played between two professional players, all the white stones would "die."

As mentioned earlier, if no handicap is given, the weaker player is given Black and plays first. Actually, this gives the stronger player (White) an automatic handicap. If the players are equal in playing strength, the player moving first will usually win by four or five points. Among equal opponents in Japan, the player who has the first move has four and a half points deducted from his score at the end of the game. If this unofficial handicap is not enough, the weaker player is allowed to put a stone on the board in a specified position and the stronger (White) plays first. (Between players of equal skill this would probably result in a loss of about nine points for the handicapped player.) Or, if this is not enough of a handicap to the stronger player, two or more stones may be placed on the board in specified positions by the weaker player, and the stronger moves first.

Amateurs can control their handicap arrangements quite easily. For example, if the weaker player wins three consecutive games with a three-stone handicap, he reduces the handicap to two stones; if he again wins three consecutive games, he reduces the handicap to one stone; if he again wins three consecutive games, he reduces the handicap to playing first. If he loses three consecutive games, however, he returns to the next higher handicap.

The specified positions for the first nine stones are shown in *Figure 9*. If the handicap is five, seven or nine stones, the last stone is placed in the center rather than in the position used when it is not the last stone in the handicap. If a handicap greater than nine stones is used, the location of the extra stones need not be dictated. However, in Japan the next four handicapped stones are placed in the positions indicated as 10, 11, 12 and 13 in *Figure 9*. The placement of these thirteen stones pertains only to handicapped games.

KO (PERPETUAL CHECK): A position such as A in *Figure 10* is called 'Ko.' The black stones enclosed by white stones on three sides will be captured by the next move on the part of White, developing into position A'. By the same token, Black may place his stone to capture the white stone and thus restore position A. Obviously White can counter with his original move—a perpetual check. To avoid such a situation, the following rule applies: If the counter move to recapture will create a possibility for a perpetual situation, that counter move cannot be made immediately after the initial capture has been effected. Thus in this example, Black must first play somewhere else on the board before he can retake—if the opportunity then exists.

SEKI* (LOCAL STALEMATE): Occasionally, a local struggle between

*Pronounced 'SAY-KEE.'

black and white pieces can develop into a draw, as shown in *Figure 11*. The area concerned is no one's territory and is omitted from the final scores in calculating territory points. Such a situation is a stalemate because if either player plays on one of the two vacant spaces, his pieces will be captured by his opponent's play on the other vacant space.

OPENING STRATEGY: Stones are placed first in the four corners and near the four edges, and the game develops toward the middle. A recollection of the primary object of GO will illustrate the reason. Since the player's aim is to obtain as large an amount of territory as possible (and only incidentally to capture his opponent's stones, which are merely a means toward

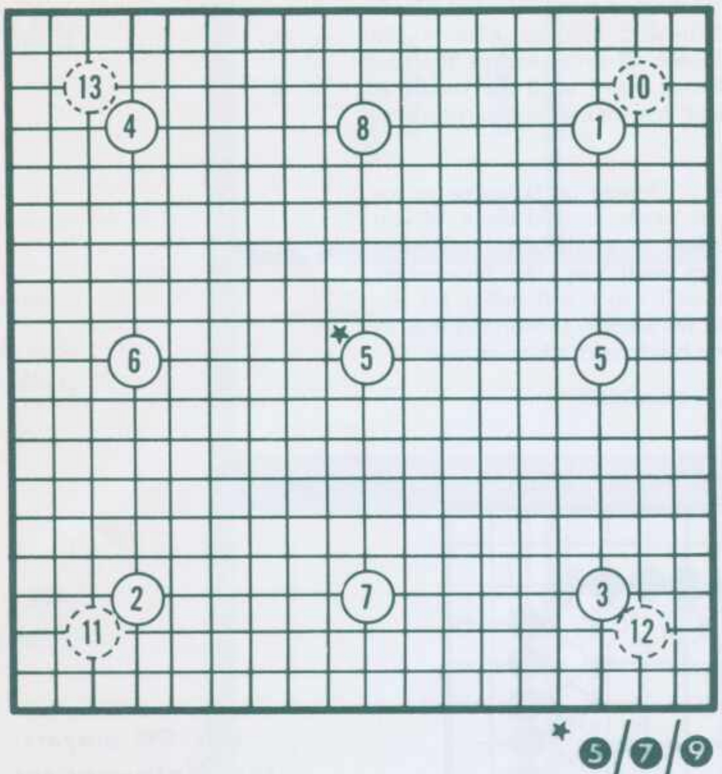


FIGURE 9

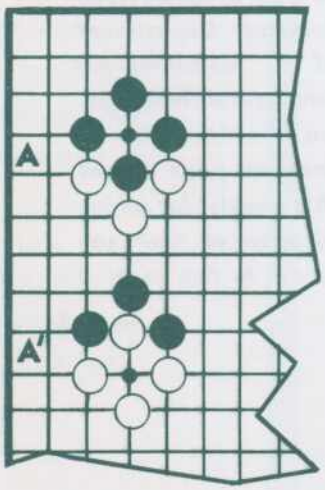
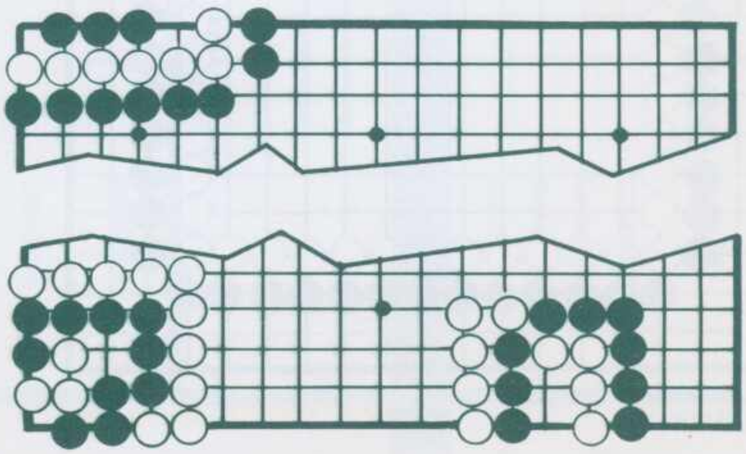


FIGURE 10

FIGURE 11



acquiring territory), his purpose is to establish lines of his own stones as boundaries to territories. The best place to do this is at the corners, the second best place at the edges. *Figure 12* illustrates this point: Black's side areas total 140 points, while White's center area counts only 121 points.

The following illustration may make this principle more fully understandable: to form one eye three stones are enough at the extreme corner, five stones at the extreme edge, and eight stones at other places. Thus the edges represent the smallest investment of stones for the same amount of territory (one point).

While there are factors other than the short-term acquisitions of territory to be considered ("influence toward the center" is one of them), development at the outside corners and edges first is the general principle to be followed. A corollary to this rule is the posting of stones in vital places on the board before settling down to hand-to-hand fighting. One of the surest signs of a novice is his tendency to ignore all areas except the one he is in. He starts out in one corner and doesn't leave until the corner is absolutely secure—by which time his opponent has staked out territory in several areas of the board.

NOTES ON GO EQUIPMENT: If there is a Chinese or Japanese sector in your city, a GO board and stones should be easily located there. If you wish to do it yourself, pencil and ruler applied to a stiff sheet of paper (eighteen inches square, with seventeen lines each way—the four edges count as the eighteenth and nineteenth lines each way) will suffice for the grid, and casino chips make a good substitute for stones. If worst comes to worst, coffee and navy beans can be used for black and white stones.

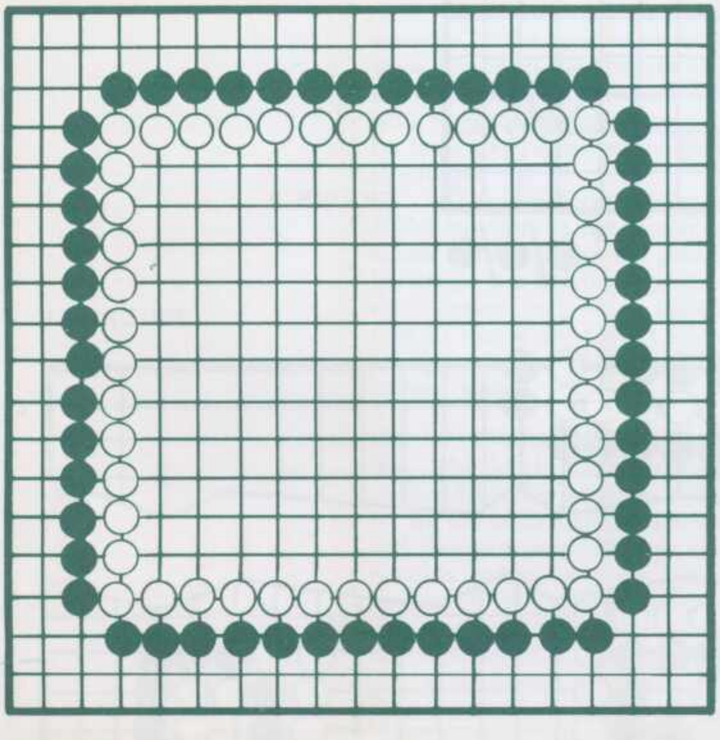


FIGURE 12

Like GO players, the Mathematical Services Department of 3C welcomes an intellectual challenge, and invites you to look at page 19 of this booklet for an indication of their capacity to rise to it.

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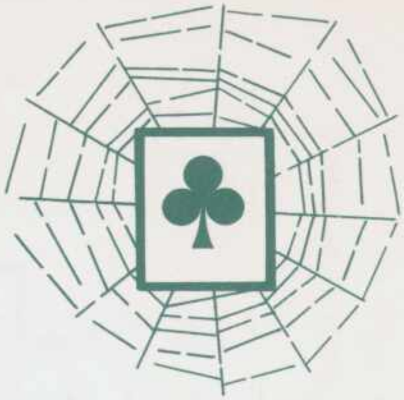
SPIDER



"Spider" is called the King of Solitaires by its fascinated addicts. Because of the large amount of conniving and higher-level juggling involved, this is no game for the after-dinner "Patience" player . . . nor should it be played in the vicinity of small children, dogs, wives, or other disturbing elements. Deceptively simple in layout, Spider affords unparalleled opportunities to mastermind a poor deal into a winning game—to display the superiority of brain over pasteboard.

Two decks shuffled together are used in the layout. Face down, deal a row of ten cards; then deal three more rows directly on top of the first row, face down. To the first four piles deal one card each, still face down. Then deal a card face up on each of the ten piles—fifty-four cards in all. The object is to get the thirteen cards of a complete suit on a pile, in order, ace uppermost (ace the surface card). Each completed suit can (but does not have to) be lifted from the pile and discarded. You win if you complete eight suits. You lose if you don't.

While the ultimate object is to complete a suit, you need not follow suit or color in building down on the piles—only numerical sequence. Thus the seven of spades, for instance, may be placed on any eight. Of course, opportunities for building "naturals" (two or more cards of the same suit in proper sequence) should by all means be taken. The surface card of any pile may be transferred to any other pile, and if it is part of a natural, all the cards may be transferred as a unit, or piecemeal. Thus a 9 8 7 6 of hearts, for example, can be moved together, separately, or as you please. Building naturals facilitates transfers. Equally as important is getting the face-down cards into action. Each time all the face-up cards of a pile are removed, turn up the next card, which can then be used in play. Spaces left by the complete removal of a pile may be filled by any available card or group of cards. This is the only way to move kings, other than building complete suits on top of them. Warning: these spaces should be filled cautiously, because they are invaluable cubbyholes in which to put annoying cards while getting suits together. Make all the plays you can with the exposed cards. When you come to a standstill, deal ten cards from your reserve face up on the piles, and working on these continue until you are stymied; then deal another ten cards. Any spaces left by the removal of an entire pile *must* be filled before dealing the next round. A new deal may block overlooked moves—eye the layout suspiciously before you make a new deal. Sometimes it is an advantage not to discard an assembled suit immediately. It can be broken up and used to bring other suits into position.

Here are a few basic pointers: Suppose that your ten piles are laid out. First, build all the naturals that you can. Then, move any card that can go on either of two others (such as an eight that could go on either of two nines); this gives you the option of moving it later if opportunity for a natural arises. Then make the remaining non-natural plays, making plays with the numerically higher cards first. After all plays have been exhausted, including those arising with the turn-up of face-down cards, deal another row of ten cards face-up. Again build naturals, cards with optional plays, and non-naturals, dealing another ten when all moves have been made, and so on, until all reserve cards (five deals) have been used. At this point you have either made your eight suits and won—or you've lost.

3C does not guarantee Spider to be nonhabit forming, nor do its mathematicians who play this game guarantee anything but the services of the Mathematical Services Department, specializing in mathematical analysis, computation, installation development services, and data processing. For interesting details, see page 19 of this booklet.

4	9	2
3	5	7
8	1	6

6	7	2
1	5	9
8	3	4

2	9	4
7	5	3
6	1	8

2	7	6
9	5	1
4	3	8

4	3	8
9	5	1
2	7	6

8	3	4
1	5	9
6	7	2

8	1	6
3	5	7
4	9	2

6	1	8
7	5	3
2	9	4

MAGIC SQUARES

A Magic Square is simply a square array of a set of numbers. What's magic about it is that these numbers—all different—are placed so that every horizontal and vertical line, as well as the two main diagonals, add up to the same sum.

Given a square array of N rows and N columns, the numbers usually played around with are the first N^2 positive integers. There are all kinds and sizes of magic squares ("panmagic," "bimagic," "multimagic," "semi-magic," etc.) but this paper will treat only one item—one simple method of generating a magic square with N being odd.

Becoming familiar with the following set of rules will enable a person to generate a magic square as large as he wants, as fast as he can move a pencil.

RULES

1. Put the number 1 in the square just below the center square.

2. Put succeeding numbers on the diagonal leading to the right and down *(Fig. 1)*. Keep following this rule whenever possible.

3. When you run out of rows (by falling out the bottom) go to the top of the column in which you want to place the number *(Fig. 2)*.

4. When you run out of columns (by falling out to the right) go to extreme left of the row in which you want to place the number *(Fig. 3)*.

5. If the space in which you wish to place a number (say X) is already occupied, put that number, X, two spaces below X-1 *(Fig. 4)*.

6. When you fall out the main diagonal going from top left to bottom right. place the next number in the extreme right of the second horizontal line; then follow rule 4, etc. *(Fig. 5)*.

The "magic number," the sum of each column, etc., is $\frac{N(N^2+1)}{2}$. The examples on page 11 were generated by this method, the "magic numbers," being 175, 369, 65.

The methods used by 3C to solve your problems may look like magic, but they're strictly mathematics. For proof, see page 19 of this booklet.

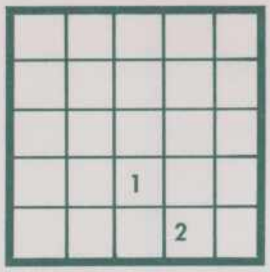


FIGURE 1

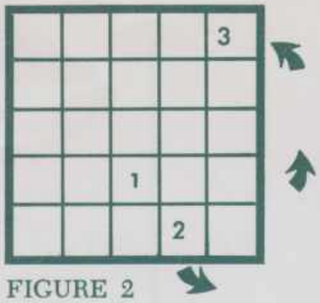


FIGURE 2

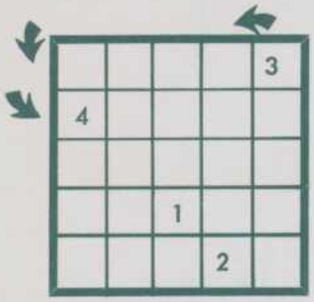


FIGURE 3

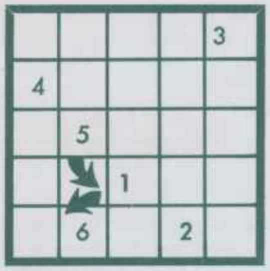


FIGURE 4

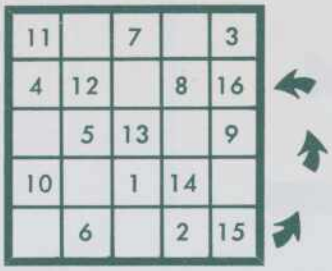


FIGURE 5

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

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11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

65

37	78	29	70	21	62	13	54	5
6	38	79	30	71	22	63	14	46
47	7	39	80	31	72	23	55	15
16	48	8	40	81	32	64	24	56
57	17	49	9	41	73	33	65	25
26	58	18	50	1	42	74	34	66
67	27	59	10	51	2	43	75	35
36	68	19	60	11	52	3	44	76
77	28	69	20	61	12	53	4	45

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NIM

If your mental gear includes a binary adder, this game should be your meat. The rules of NIM are so simple that you will have no trouble finding opponents—until you have achieved the reputation of being unbeatable. The game is frequently played in bars, using coasters or matches as counters, with the loser buying the next round; or on the beach, using pebbles, with sea shells at stake.

NIM is played by two people playing alternately. Before the play starts, an arbitrary number of chips or objects is put into an arbitrary number of piles, in any distribution whatever. Then each player in his turn removes as many chips as he wishes from any pile—but from only one pile, and at least one chip. The player who takes the last chip is the winner. There is only one secret to winning this game, and that is: "Always present your opponent with an 'even position.'"

To determine whether a given configuration of piles and chips is an even position or not, you must:

1. *Represent the number of chips in a pile in binary notation; do this for all piles.*

2. *Add the digits in the units position of these binary representations, with no carry into next column; do the same for each binary "place" or column.*

If the sum of each column is divisible by 2, it is an even position; otherwise it isn't.

For example, 4 piles consisting of 3, 7, 13, and 15 would be calculated as in *Figure 1.* This is obviously not an E P (even position). If it were our turn to play, we would change this layout so that the column sums were either 2-2-2-4 or 2-2-4-4. This could be done in any one of the following ways:

a) Remove 6 from "7" pile *(Figure 2).*
b) Remove 6 from "15" pile *(Figure 3).*
c) Remove 2 from "13" pile *(Figure 4).*

3 =	1 1
7 =	1 1 1
13 =	1 1 0 1
15 =	1 1 1 1
	2 3 3 4

FIGURE 1

3 =	1 1
7-6 = 1 =	1
13 =	1 1 0 1
15 =	1 1 1 1
	2 2 2 4

FIGURE 2

3 =	1 1
7 =	1 1 1
13 =	1 1 0 1
15-6 = 9 =	1 0 0 1
	2 2 2 4

FIGURE 3

3 =	1 1
7 =	1 1 1
13-2 = 11 =	1 0 1 1
15 =	1 1 1 1
	2 2 4 4

FIGURE 4

It is one of these configurations that we want to leave for our opponent after we have moved, and a simple method for determining how many chips to remove from which pile will be explained.

To calculate the correct move we determine which columns are odd and which, therefore, must be changed. We then select any pile which has a *one* in the leftmost column needing change, and mentally change the bits only in those columns needing change, replacing ones by zeros and zeros by ones; this new number will be smaller than the original and it is the number of chips we want to leave in that pile after we have moved. Referring back to the previous example, you can see that this is what was done. There is good reason why this strategy is a winning one. From an E P it is impossible to move to another E P; and from a non-E P, it is always possible to move to an E P, and in general possible to move to a non-E P. And since the final winning move consists of presenting our opponent with zero chips in every pile (which is an even position) all we have to do is give him an E P before he gives us one, and then "maintain the tempo." (All of the above can be proved mathematically.)

Sometimes the game is played with the rule: The person removing the last chip is the loser. If you are playing this rule, the strategy is exactly the same up to the point where, *for the first time, you would leave only one chip in every non-empty pile*. At this very step you should leave zero chips in the pile you are changing if you were going to leave one chip, and vice-versa.

EXAMPLE (Our turn to move):

Rule: Last chip taker is winner.

No. 1. Piles left are 1, 1, 1, N; where N is any integer greater than 1. Correct move is to remove N-1 chips from the N-pile.

No. 2. Piles left are 1, 1, N; we should remove N chips.

Rule: Last chip taker is loser.

No. 3. Piles left are 1, 1, 1, N; we will remove N chips and be a winner.

No. 4. Piles left are 1, 1, N; we take N-1 chips away.

These are the procedures to follow if your opponent has not presented *you* with an even position at the beginning of the game. In that case you have a problem, and your only strategy is to hope that your opponent misplays.

FIGURE 1

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

FIGURE 2

1	2	3	4
5	6	7	8
9	10	11	12
13	15		14

	15	14	13
12	11	10	9
8	7	6	5
4	3	2	1

FIGURE 3

15	14	13	12
11	10	9	8
7	6	5	4
3	2	1	

FIGURE 4

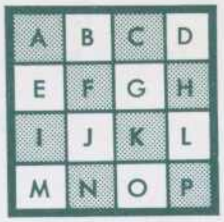

FIGURE 5

THE 15 PUZZLE

The "15 Puzzle" consists of the numbers from 1 to 15 and one blank arranged in a square array *(Figure 1).* The only rule is this: Any one of the numbers to the immediate right, left, top or bottom of the blank space can be moved into the empty space. In *Figure 1,* only 15 or 12 can be moved. The problem posed by the puzzle is: given some other array as a "goal," (for example, *Figures 2, 3 or 4*) *can the numbers in the* ORIGINAL CONFIGURATION OF FIGURE 1 be so maneuvered as to achieve the goal?

This puzzle (known also as the "Boss Puzzle" and "le Jeu de Taquin") has been played for many years. After its appearance in 1878 Europe went 15-Puzzle mad. The game was played in the streets, factories, salons, and palaces by every level of society. Tournaments were held. Business houses posted notices forbidding playing of the game during working hours under penalty of dismissal. Enormous sums of money were offered for the solution of certain apparently simple problems. (These prizes were never collected, for a very good reason: the solutions were mathematically impossible.) In the 1930's some genius invented a contraption whereby the numbers could be moved horizontally or vertically as per the rules, but could not be removed from the base holding the numbers, thus foiling all would-be cheaters.

With the convention that the empty square is counted as number 16, there are 16 factorial different arrays of the numbers, and exactly one half of these—10,461,394,944,000—are impossible goals. There is an easy method for determining whether a given goal is possible or not. Skipping the derivation of this method with its roots in the theory of permutations, the method is simply this:

1. First consider the individual squares (the positions) as having permanent names as shown in *Figure 5*.

2. Consider the number (call it X) in position A of the goal, and count how many numbers smaller than X are sitting in positions higher-lettered than A. For *Figure 2* this would be 0; *Figure 3* (counting the blank as 16), 15; and *Figure 4*, 14.

3. Do this for all the positions, and add up the numbers that were obtained for positions A through P.

4. If the blank square is in one of the unshaded squares of *Figure 5*, add one to the sum; if in a shaded square, the sum is left alone.

5. If this final sum is even, the goal is possible; if odd, impossible.

Example, using *Figures 2, 3 and 4* as goals:

	Number of Smaller Numbers Following		
Position	In Fig. 2	In Fig. 3	In Fig. 4
A	0	15+0=15	14
B	0	14	13
C	0	13	12
D	0	12	11
E	0	11	10
F	0	10	9
G	0	9	8
H	0	8	7
I	0	7	6
J	0	6	5
K	0	5	4
L	0	4	3
M	0	3	2
N	1	2	1
O	1+1=2	1	0
P	0	0	0+0=0
TOTAL	3	120	105

Thus *Figure 3* is possible to achieve, *Figures 2* and *4* are not.

If when presented with a goal, you can add up these numbers mentally, or just calculate the parity, and come up with an immediate "Impossible" you might be able to get some fast bets down. This is the easy case as the burden of proof lies on the other person; be generous, give him a week. If the sum is even, and you have made the claim of "Sure it's possible" then of course you'll have to prove it, and *you* will have to push the numbers around.

If you have purchased a puzzle on which the original configuration is Figure 2, reverse "possible" and "impossible" throughout this exposé.

Which reminds us: if you have problems which involve pushing numbers around and you are doubtful that your goal can be achieved, turn to page 19 of this booklet for a solution.

KRIEGSPIEL

Kriegspiel (War Game*) is a variant of chess which is unexcelled for action, unconscionable (but very legal) kibitzing, and preposterous situations. The play is more closely related to military tactics than to chess strategy, for neither player knows the actual position of his opponent's men. With the addition of spectators, Kriegspiel takes on some of the qualities of a final inning in the Yankee Stadium.

The literature on Kriegspiel is exceedingly small. There is a mention of a Kriegs-schachsspiel (War Chess) invented and in vogue in the eighteenth century, which may or may not be the ancestor of Kriegspiel. In the West, the game has been popularized by and is largely identified with RAND Corporation of Santa Monica, where it is staple lunchtime recreation. While this corporation, contrary to popular opinion, did not invent the game (it was introduced there by emigrant mathematicians from Princeton a decade ago), it has influenced and defined the play, and the laws we shall use here are for the most part those of Kriegspiel as played at RAND.**

THE GAME is played with standard chess rules, two players, a referee, and kibitzers—whose role will be discussed later. Each player has a complete chess set (board, black and white men). The players may not see each other's boards and men (an upended drawer or piece of cardboard separating the players will accomplish this; or they can sit back to back). The referee seats himself so as to see both boards simultaneously, and duplicates on his board (if used) the moves of each player. His board should be screened or otherwise not visible to the players. Although tyros will find that using a third chess set eliminates much error (the beginning referee makes more mistakes than the beginning player), an experienced referee obtains a faster, less complicated game by visualizing the positions.

EACH PLAYER sets up his men as in an ordinary game. He must always keep the position of his own forces correct on the board, but is free at any time to arrange or rearrange the enemy men in the way he infers, guesses or hopes his opponent has placed them. When a player makes a capture, he should always remove one of the enemy's men from his board. He may be uncertain of what he has netted, but in this manner keeps count of the number of enemy men left. The player is responsible for keeping track of his captures, and if he loses count, so much the worse for him. "It is considered ethical for a player to capitalize on blunders and all unsolicited information received from the referee and kibitzers; he may solicit 'information' from his opponent or otherwise heckle."***

THE REFEREE is most important to Kriegspiel. He announces:

1. Moves: He informs each player when the other has completed his move, but does not state what it is.

**A game used for the instruction of military tactics, employing metal counters on a miniature battlefield or naval chart, also goes under this name. A Prussian invention of 1824, it came into extensive use by our Army and Navy after the Civil War.*

***While the game is currently played not only in the United States but in Europe, there are no known official Kriegspiel rules.*

****Quoted from a RAND memo on the subject.*

A move is completed when a man touches the board or a presumed enemy man on a legally admissible square (as differentiated from the chess rule, in which a move is not completed until the player takes his hand away from the man).

2. Illegal Tries: He announces each false try with a "No." False tries, or rebuffs (those which are not legal, as moving the King into check, or impossible, as moving a pawn forward to a square occupied by the enemy) do not constitute moves. There is no limit to the number of false tries made. A false try with one man does not obligate the player to use the same man to complete the move. When a player makes a move which is patently illegal (as moving a pawn diagonally when no pawn try has been announced, or moving to or through a square occupied by one of his own men) the ordinary false-try "No" is replaced by a special pronouncement: "Hell, no!"

3. Pawn Tries: He announces on which squares the mover's pawns have opportunities to capture. (This is an expedient which avoids the time-consuming process of having the player move many or even all of his pawns diagonally at each move to discover possible captures.) The player need not take advantage of these announcements. A double pawn try, in which two pawns have opportunities to capture on the same square, is not specified as such by the referee. As in a single pawn try, he announces only the square on which a man may be captured. En passant options are not specified as such; they are indicated only as ordinary pawn-capture options.

4. Captures: He announces captures, the square on which they occurred, and whether the man taken was a Pawn or a Piece. Thus the player who has been captured does not know what attacked him; his opponent does not know (unless it is a Pawn) what he has snagged.

5. Checks: He announces checks and their direction, indicating whether on:

a. A file (vertically)
b. A rank (horizontally)
c. A long diagonal
d. A short diagonal
e. Or by a Knight

The long and short diagonals are referential to the King. For example, "check on the short diagonal" would describe a check (by a Pawn, Bishop or Queen) on the shorter of the *two diagonals which intersect at the King*. When a check exists, only those Pawn options to capture which will eliminate the check are announced. A Pawn check is not specified as such; it is announced only as a check on the long or short diagonal.

6. Pawn Promotions: He announces the promotion of a Pawn, but not the square at which it occurred nor the piece to which it was promoted.

7. Checkmate, Stalemate and Drawn Game: Castling is governed by the usual laws.

A player may, before moving, or even before his opponent has completed his move, ask the referee for a recapitulation of the rebuffs (false tries) made by the enemy during his last move.

The referee's decision on all matters is final, whether it concerns dispute over the position of the men or declaring the game void. He may use either of two methods of designating the squares. One is to mark each square with a number (from one to sixty-four).* The other is, as in ordinary chess notation, to designate the square with reference to the mover; thus if it is announced that White captures on his King-Bishop eight, Black notes this as occurring on his King-Bishop square.

The referee corrects any error in the quantity or quality of the player's own men (as when the player omits removing his own captured man). He tries to eliminate such errors as the player's misidentification of a square named in an announcement, if it can be done without giving away significant information. If he makes an error, the game can be reversed for a few moves to correct it, or if the error is far-reaching enough, the game may be declared void.

KIBITZERS are an integral part of Kriegspiel. "The game is a spectator sport par excellence, and everything is done to keep it so. The kibitzers have the right to criticize the play, the players and the referee. However, the ethics of the situation require that the kibitzers never intentionally give useful information to the players."** Inversely, any kibitzer who proffers erroneous advice or comment is simply adhering to the Kriegspiel sporting code. The player who receives such gratuitous enlightenment as the information that he has just captured a Knight would do well to look his gift horse in the mouth.

THE STRATEGY of Kriegspiel is, as has been implied, unlike that of chess. The essence is to deduce, by logical inference, where the enemy's men are, and it does not follow that a good chess player makes a good Kriegspiel player. A player with little more than a working knowledge of chess can, if he is opportunistic and keen enough, make hash of a good (but cautious) chess player.

There are various ways in which the player can gather information about the enemy. The announcement of a capture of course locates the position of the attacking piece; and the player, by putting two and two together, may be able to identify the piece. The player can make long-range moves of the Rook, Queen or Bishop to determine if a file, rank or diagonal is open, or blockaded (indicated by the referee's "No") by the enemy.

Another ploy is to walk the King forward to discover by illegality what squares the enemy commands. This is especially good in the end game. If an attempted move is met with a rebuff, the player knows that the enemy is attacking the square to which the King was moved on a false try. Since the King cannot be moved to an attacked square (as opposed to another man, where the move would be legal and the piece might be captured) he is an excellent tool for determining the deployment of enemy men. (The enemy, on the other hand, may gather by the repeated "No's" that the King is out and attempt to mate him.)

If the player parries a check by interposing a man, he can discover later whether the checking piece is still there by moving the (presumably) pinned man.

When in check, the King should be used in an attempt to remove the checking man; for example, if a check is announced on a diagonal (or file, or rank), the King, before any other move, should try both adjacent squares in an effort to capture his attacker. If the enemy has only a hazy idea of the King's position and has not supported his man, this may result in a coup. When a capture has been made, the attacking piece should either be moved or supported.

**A method of numbering the squares which makes them very easy to locate is not in numerical sequence but by the file. Thus White's Queen-Rook square would be No. 11 (file 1, square 1); the Rook-Pawn square, No. 12 (file 1, square 2); the Knight square, No. 21 (file 2, square 1); the Knight-Pawn square, No. 22 (file 2, square 2); the Bishop square, No. 31 (file 3, square 1), and so on. The left Rook file would then extend from 11 to 18; the right Rook file, of course, from 81 to 88.*

***Quoted from a RAND memo on the subject.*

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